MATH 7A Test 4 – SAMPLE

This test is in two parts. On part one, you may not use a calculator; on part two, a (non-graphing) calculator is necessary. When you complete part one, you turn it in and get part two. Once you have turned in part one, you may not go back to it. You will show all work on the test paper, no scratch paper is allowed.

PART ONE - NO CALCULATORS ALLOWED

(1) Find each of the following exactly. If is a trig value we cannot compute exactly, write "can't find exactly value":

(a) cos (330°) = $\sqrt{3}/2$

(b) sin ($-\pi/2$) = -1

(c) tan ($3\pi/2$) = undefined

(d) tan (120°) = $-\sqrt{3}$

(e) tan (160°) = can't find exactly

(f) csc($3\pi/4$) = $\sqrt{2}$

(g) cos ($7\pi/6$) = $-\sqrt{3},2$

(h) sec (225°) = $-\sqrt{2}$

(i) sin (-120°) = $-\sqrt{3}/2$

(j) tan ($-\pi/3$) = $-\sqrt{3}$

(k) cos (180°) = -1

(l) sin ($\pi/8$) = can't find exactly

(m) tan $\left(-\frac{\pi}{2}\right)$ = undefined

(n) cos (315°) = $\sqrt{2}/2$

(o) sin (1) = can't find exactly

(p) cot $\left(\frac{\pi}{3}\right)$ = $\frac{1}{\sqrt{3}}$

(q) cos $\left(\frac{4\pi}{3}\right)$ = $-1/2$

(r) cos (390°) = $\sqrt{3}/2$

(s) sin (-150°) = $-1/2$

(t) csc $\left(-\frac{\pi}{4}\right)$ = $-\sqrt{2}$

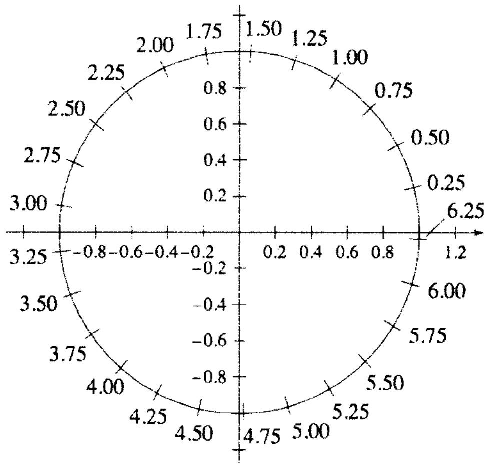
(3) Use the figure to (4 points)

(a) approximate the value of cos 1 ≈ 0.5 and sin 5.5 ≈ -0.7

(b) find a value of t such that sint $\approx$ -0.2 3.3 or 6.1

(approx)

(c) find a value of t such that cost $\approx$ 0.6 0.8 or 5.3



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MATH 7A Test 4 sample

PART TWO - CALCULATORS ALLOWED (non-graphing)

Show your work on this paper. EXACT answers are expected unless otherwise specified. Show scales on graphs. Give units in answers when appropriate.

Fill in the blanks.

(1) $f(t) = \sin(t)$ Is even, odd, or neither ___odd___

(2) Convert to degrees (exactly) $7\pi/9$ radians___$140°$___

$\nearrow \frac{\pi}{4}$ $s = r\theta$

(3) What is the length of the arc that subtends a central angle of 45° in a circle of radius 8 inches? ___2π inches___

(4) In which quadrant(s), if any, is $\sin\theta < 0$ and $\cos\theta > 0$ ___4th___

(5) Convert to radians (exactly) $100°$ ___$5\pi/9$ radians___

(6) Using your calculator, find approximations for the following, correct to 3 decimal places.

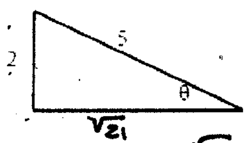
(a) $\sec 22° \approx$ ___1.079___

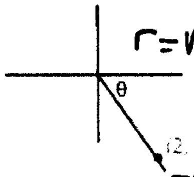
(b) $\tan(-\pi/7) \approx$ ___-0.482___

(c) $\frac{2\cos 60°}{\sin 10° + 7} \approx$ ___0.139___

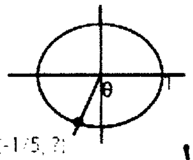
(d) $\cos 2 \approx$ ___-0.416___

(7) Given the following figures, find:





$r = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$



$x^2 + y^2 = 1$

$(-\frac{1}{5})^2 + y^2 = 1$

$y^2 = \frac{24}{25}$

$y = \pm\sqrt{\frac{24}{25}}$, Q3

$y = \frac{-2\sqrt{6}}{5}$

(a) $\cos\theta =$ ___$\sqrt{21}/5$___

(b) $\sin\theta =$ ___$\frac{-4}{2\sqrt{5}} = \frac{-2}{\sqrt{5}}$___

(c) $\cot\theta =$ ___$\frac{1}{2\sqrt{6}}$___

(8) A ferris wheel at the carnival has a radius of 30 feet. You measure the time it takes for one revolution to be 70 seconds. Find the following exactly. Give units!

a) what is the angular speed?

b) what is the linear speed?

c) what is the linear speed in miles per hour? (Find exact value, then calculate an approximate answer using your calculator.)

a) $\frac{1\text{ rev}}{70\text{ sec}} \cdot \frac{2\pi\text{ rad}}{\text{rev}} = \boxed{\frac{\pi}{35}\frac{\text{rad}}{\text{sec}} = \omega}$

b) $V = r\omega = 30 \cdot \frac{\pi}{35} = \boxed{\frac{6\pi}{7}\text{ ft/sec}}$

c) $\frac{6\pi}{7}\frac{\text{ft}}{\text{sec.}} \cdot \frac{1\text{ mile}}{5280\text{ ft}} \cdot \frac{3600\text{ sec}}{\text{hr}} = \frac{21{,}600\pi}{36{,}960} = \boxed{\frac{45\pi}{77} \approx 1.84\text{ miles/hr.}}$

(9) For each of the following angles, find the quadrant and reference angle. Answer exactly, in the units given.

	ANGLE	QUADRANT	REFERENCE ANGLE
a)	390°	I	30°
b)	-210°	II	30°
d)	$7\pi/6$	III	$\pi/6$
e)	$19\pi/10$	IV	$\pi/10$
g)	2.5	II	$\pi - 2.5$

(10) Find 4 angles, one in each quadrant, having the given angle as a reference angle.

$2\pi/9$ $\quad 2\pi/9 \quad 7\pi/9 \quad 11\pi/9 \quad 16\pi/9$

(11) Name four "$\pi/6$-type" angles, as defined in class.

$\pi/6 \quad 5\pi/6 \quad 7\pi/6 \quad 11\pi/6$

(12) Find 2 angles coterminal with the given angle, one positive, one negative.
(Answer in the units given)

(a) 170° $\quad 530^\circ, -190^\circ$

(b) $23\pi/12$ $\quad -\frac{\pi}{12}, \frac{47\pi}{12}$

(13) Find the missing coordinate of P (_______, 1/7) using the fact that P lies on the unit circle in the second quadrant.

$x^2+y^2=1$ $\quad x^2+\frac{1}{49}=1$ $\quad x^2=\frac{48}{49}$ $\quad x=\pm\sqrt{\frac{48}{49}}=\pm\frac{4\sqrt{3}}{7}$

In Quad 2, $x<0$ so $x=\frac{-4\sqrt{3}}{7}$

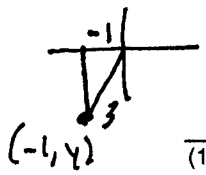
(14) Find the terminal point P(x,y) on the unit circle determined by $t=\frac{5\pi}{4}$.

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

(15) Given $\cos\theta = \frac{-1}{3}$ and θ is in Quadrant III, find:

(a) $\sin\theta = \frac{-2\sqrt{2}}{3}$

(b) $\tan\theta$ $\quad 2\sqrt{2}$



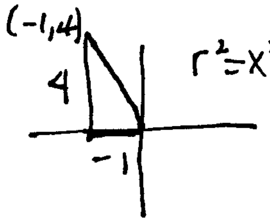
$(-1)^2+y^2=3^2$

$y=-\sqrt{8}=-2\sqrt{2}$

(16) Given $\tan\theta = -4$ and $\cos\theta < 0$ find:

(a) $\sin\theta = \frac{4}{\sqrt{17}}$

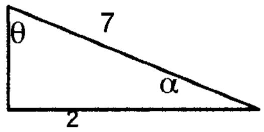
(b) $\cot\theta$ $\quad -\frac{1}{4}$



$r^2=x^2+y^2=\sqrt{17}$

(17) Given the following right triangle, find cosα, , tanθ exactly

$\sqrt{49-4} = \sqrt{45}$
$= 3\sqrt{5}$



cosα= $\frac{2}{7}$ tanθ= $\frac{2}{3\sqrt{5}}$

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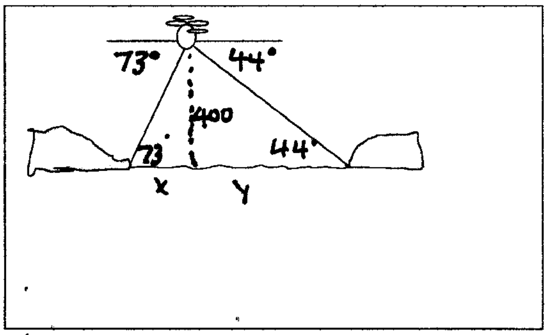
(18) A helicopter hovers 400 feet above a river. The angle of depression from the helicopter to the west bank is 73°, while the angle of depression from the helicopter to the east bank is 44°. (Remember, angle of depression is measured from the line of sight to the horizontal). Find the width of the river (exact and approximate.)



width of river $= x + y$

$= \frac{400}{\tan 73°} + \frac{400}{\tan 44°}$ (exact)

$= 536.5$ ft

$\tan 73° = \frac{400}{x}$ $\tan 44° = \frac{400}{y}$

$x = \frac{400}{\tan 73°}$ $y = \frac{400}{\tan 44°}$

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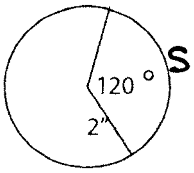
(19) Find the length of the arc and the area of the sector corresponding to a central angle of 120 degrees and radius 2 inches..



need θ in radians

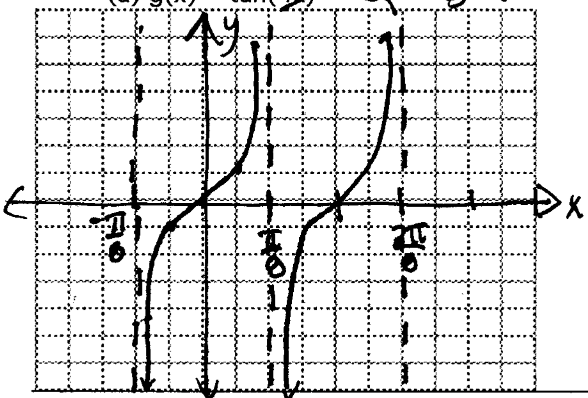
$120° \cdot \frac{\pi}{180°} = \frac{2\pi}{3}$ radians

Arclength $s = r\theta = 2 \cdot \frac{2\pi}{3} = \frac{4\pi}{3}$ inches

Area $= \frac{1}{2} r^2 \theta = \frac{1}{2}(2)^2 \frac{2\pi}{3} = \frac{4\pi}{3}$ sq. inches not 2π

(20) Sketch the following graphs. (clearly show scale, graph at least two periods, clearly show locations of asymptotes)

(a) $g(x) = \tan(4x)$ period $= \frac{2\pi}{8} = \frac{\pi}{4}$



Period is $\frac{\pi}{\omega} = \frac{\pi}{4}$

Find an asymptote:

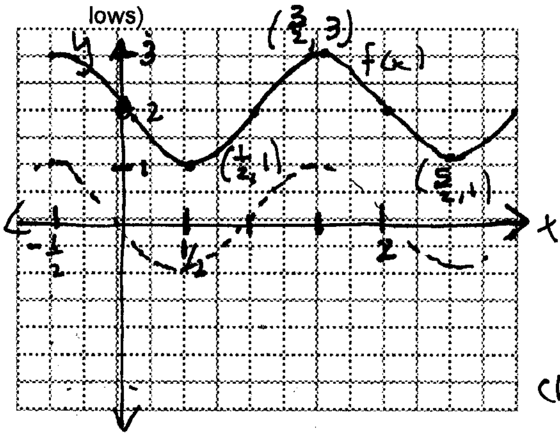
$\tan 4x = \frac{\sin 4x}{\cos 4x}$

Asymptote when $\cos 4x = 0$

so $4x = \frac{\pi}{2}$

$x = \frac{\pi}{8}$

(b) $f(x) = 2 - \sin \pi x$ (clearly show scale, graph at least one period, label coordinates of highs and lows)



Period $= \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

$\frac{1}{4}$ period $= \frac{1}{4} \cdot 2 = \frac{1}{2}$ is good scale

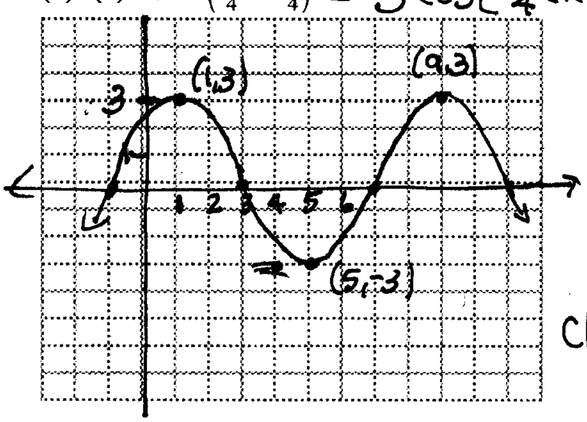
Graph $y = \sin \pi x$

then flip

then shift up 2

check a point

(c) $f(x) = 3\cos\left(\frac{\pi}{4}x - \frac{\pi}{4}\right) = 3\cos\left(\frac{\pi}{4}(x-1)\right)$



Period $= \frac{2\pi}{\pi/4} = 8$

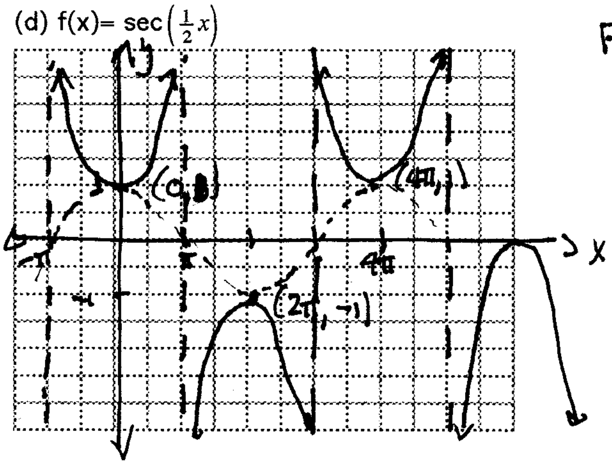
$\frac{1}{4}$ period $= 2$

Shift right 1

1 square = 1 unit

Check a point

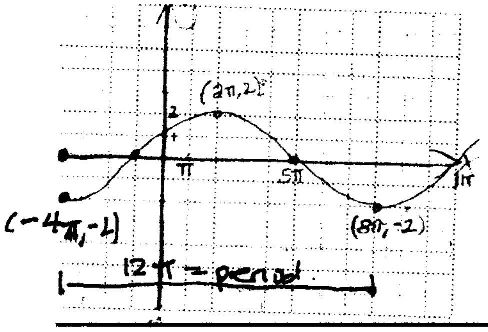
(d) $f(x) = \sec\left(\frac{1}{2}x\right)$



Period $= \frac{2\pi}{\omega} = \frac{2\pi}{1/2} = 4\pi$ $\frac{1}{4}$ period $= \pi$

graph $y = \cos \frac{1}{2}x$ then use reciprocal idea

(21) Find an equation corresponding the graph below. Check a point. (5 points)



period $= 12\pi$ so $\frac{2\pi}{\omega} = 12\pi \Rightarrow \omega = \frac{1}{6}$

$y = 2\sin\left(\frac{1}{6}(x+\pi)\right)$ $\quad y = 2\sin\left(\frac{1}{6}\pi + \frac{\pi}{6}\right)$

$y = 2\cos\left(\frac{1}{6}(x-2\pi)\right)$ $\quad y = 2\cos\left(\frac{1}{6}x - \frac{\pi}{3}\right)$ check a point

(others)

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(22) Consider a ball that is bouncing up and down on the end of a spring in simple harmonic motion. Suppose that 4 inches is the maximum distance the ball moves vertically upward or downward from its equilibrium position. Suppose also that the time it takes for the ball to complete one cycle is 6 seconds. Find an equation for the motion of the ball in each of the cases below.

(6 points)

a) The ball starts at rest at t=0 and is pushed downward as t increases.

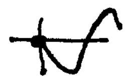
b) The ball is pushed upward to its maximum displacement ~~below~~ above equilibrium and is let go at time t=0.

period = 6 seconds

$\frac{2\pi}{\omega} = 6$

$\omega = \frac{\pi}{3}$

so $d = \pm 4 \begin{smallmatrix}\sin\\ \cos\end{smallmatrix}\left(\frac{\pi}{3}t\right)$

a)  "−sine" $\quad d = -4\sin\frac{\pi}{3}t$

b)  "+cos" $\quad d = 4\cos\frac{\pi}{3}t$

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(22) If a tower is viewed from the top of a 50 foot building, the angle of elevation to the top of the tower is 59°. If viewed from the ground floor of the same building, the angle of elevation to the top of the tower is 62°. Find the height of the tower. (Show an exact answer and an approximate.)

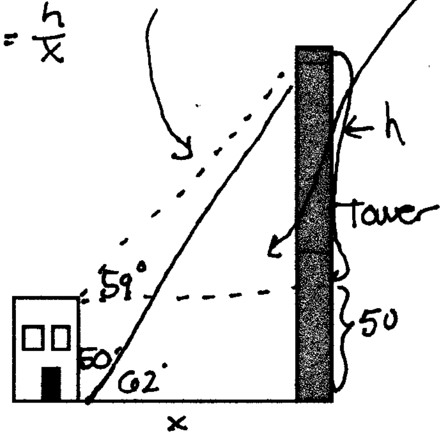
(15 points)

smaller right triangle

$\tan 59° = \frac{h}{x}$

larger right Δ

$\tan 62° = \frac{h+50}{x}$



$$\begin{cases} x\tan 59° = h \\ x\tan 62° = h+50 \end{cases} \Rightarrow \begin{cases} x = \frac{h}{\tan 59°} \\ x = \frac{h+50}{\tan 62°} \end{cases}$$

$$\frac{h}{\tan 59°} = \frac{h+50}{\tan 62°}$$

$h\tan 62° = (h+50)\tan 59°$

$h\tan 62° = h\tan 59° + 50\tan 59°$

$h\tan 62° - h\tan 59° = 50\tan 59°$

$h(\tan 62° - \tan 59°) = 50\tan 59°$

$h = \frac{50\tan 59°}{\tan 62° - \tan 59°}$

ht. of tower $= h+50 = \frac{50\tan 59°}{\tan 62° - \tan 59°} + 50 \approx 434.5$ ft